gk 12m2 — 1. Stegreifaufgabe aus der Mathematik — 25.10.2007 — Gruppe A

Lösungsvorschlag

1. $\int_{-1}^{2}\left(\frac{1}{6}x^3-\frac{1}{4}x\right)dx = \frac{1}{6}\cdot\left(\frac{2^4}{4}-\frac{(-1)^4}{4}\right)-\frac{1}{4}\cdot\left(\frac{2^2}{2}-\frac{(-1)^2}{2}\right) =$

$= \frac{1}{6}\cdot\left(4-\frac{1}{4}\right)-\frac{1}{4}\cdot\left(2-\frac{1}{2}\right) = \frac{1}{6}\cdot\frac{15}{4}-\frac{1}{4}\cdot\frac{3}{2} = \frac{5}{8}-\frac{3}{8} = \frac{1}{4}$

2. $\int_{a}^{b}(2-\sqrt{x})dx-\int_{c}^{a}(\sqrt{x}-2)dx-\int_{b}^{c}(\sqrt{x}-2)dx =$

$= \int_{a}^{b}(2-\sqrt{x})dx+\int_{c}^{a}(2-\sqrt{x})dx+\int_{b}^{c}(2-\sqrt{x})dx =$

$= \int_{a}^{c}(2-\sqrt{x})dx-\int_{a}^{c}(2-\sqrt{x})dx = 0$

3. $\int_{0}^{2}(f(x)-g(x))dx = \int_{0}^{2}\left(2x^2+1+\sqrt{x}-\left(\frac{1}{2}x+\sqrt{x}\right)\right)dx = \int_{0}^{2}\left(2x^2+1-\frac{1}{2}x\right)dx =$

$= 2\cdot\left(\frac{2^3}{3}-\frac{0^3}{3}\right)+\left(\frac{2^1}{1}-\frac{0^1}{1}\right)-\frac{1}{2}\cdot\left(\frac{2^2}{2}-\frac{0^2}{2}\right) = 2\cdot\frac{8}{3}+2-\frac{1}{2}\cdot 2 = \frac{16}{3}+1 = \frac{19}{3}$

gk 12m2 — 1. Stegreifaufgabe aus der Mathematik — 25.10.2007 — Gruppe B

Lösungsvorschlag

1. $\int_{-2}^{1}\left(\frac{1}{5}x^2-\frac{1}{2}x^3\right)dx = \frac{1}{5}\cdot\left(\frac{1^3}{3}-\frac{(-2)^3}{3}\right)-\frac{1}{2}\cdot\left(\frac{1^4}{4}-\frac{(-2)^4}{4}\right) =$

$= \frac{1}{5}\cdot\left(\frac{1}{3}-\frac{-8}{3}\right)-\frac{1}{2}\cdot\left(\frac{1}{4}-4\right) = \frac{1}{5}\cdot\frac{9}{3}-\frac{1}{2}\cdot\frac{-15}{4} = \frac{3}{5}-\frac{-15}{8} = 2{,}475 = 2\frac{19}{40} = \frac{99}{40}$

2. $\int_{a}^{c}(1-\sqrt{x})dx-\int_{c}^{b}(\sqrt{x}-1)dx-\int_{b}^{a}(\sqrt{x}-1)dx =$

$= \int_{a}^{c}(1-\sqrt{x})dx+\int_{c}^{b}(1-\sqrt{x})dx+\int_{b}^{a}(1-\sqrt{x})dx =$

$= \int_{a}^{b}(1-\sqrt{x})dx-\int_{a}^{b}(1-\sqrt{x})dx = 0$

3. $\int_{0}^{2}(f(x)-g(x))dx = \int_{0}^{2}\left(1+3x^2-\sqrt{x}-\left(\frac{3}{2}x-\sqrt{x}\right)\right)dx = \int_{0}^{2}\left(1+3x^2-\frac{3}{2}x\right)dx =$

$= \left(\frac{2^1}{1}-\frac{0^1}{1}\right)+3\cdot\left(\frac{2^3}{3}-\frac{0^3}{3}\right)-\frac{3}{2}\cdot\left(\frac{2^2}{2}-\frac{0^2}{2}\right) = 2+8-3 = 7$

gk 12m4 1. Stegreifaufgabe aus der Mathematik 25.10.2007 Gruppe A
Lösungsvorschlag

1. $\int_{-2}^{2}(x-3)^2dx = \int_{-2}^{2}(x^2-6x+9)dx = \left(\frac{2^3}{3}-\frac{(-2)^3}{3}\right)-6\cdot\left(\frac{2^2}{2}-\frac{(-2)^2}{2}\right)+9\cdot\left(\frac{2^1}{1}-\frac{(-2)^1}{1}\right) =$
$= \left(\frac{8}{3}+\frac{8}{3}\right)-6\cdot 0+9\cdot(2+2) = \frac{16}{3}+36 = \frac{124}{3} = 41\frac{1}{3}$

2. $\int_a^b(2-\sqrt{x})dx-\int_c^a(\sqrt{x}-2)dx-\int_b^c(\sqrt{x}-2)dx =$
$= \int_a^b(2-\sqrt{x})dx+\int_c^a(2-\sqrt{x})dx+\int_b^c(2-\sqrt{x})dx =$
$= \int_a^c(2-\sqrt{x})dx-\int_a^c(2-\sqrt{x})dx = 0$

3. $\int_0^2(f(x)-g(x))dx = \int_0^2\left(2x^2+1+\sqrt{x}-\left(\frac{1}{2}x+\sqrt{x}\right)\right)dx = \int_0^2\left(2x^2+1-\frac{1}{2}x\right)dx =$
$= 2\cdot\left(\frac{2^3}{3}-\frac{0^3}{3}\right)+\left(\frac{2^1}{1}-\frac{0^1}{1}\right)-\frac{1}{2}\cdot\left(\frac{2^2}{2}-\frac{0^2}{2}\right) = 2\cdot\frac{8}{3}+2-\frac{1}{2}\cdot 2 = \frac{16}{3}+1 = \frac{19}{3}$

gk 12m4 1. Stegreifaufgabe aus der Mathematik 25.10.2007 Gruppe B
Lösungsvorschlag

1. $\int_{-2}^{2}(x+2)^2dx = \int_{-2}^{2}(x^2+4x+4)dx = \left(\frac{2^3}{3}-\frac{(-2)^3}{3}\right)+4\cdot\left(\frac{2^2}{2}-\frac{(-2)^2}{2}\right)+4\cdot\left(\frac{2^1}{1}-\frac{(-2)^1}{1}\right) =$
$= \left(\frac{8}{3}+\frac{8}{3}\right)-4\cdot 0+4\cdot(2+2) = \frac{16}{3}+16 = \frac{64}{3} = 21\frac{1}{3}$

2. $\int_a^c(1-\sqrt{x})dx-\int_c^b(\sqrt{x}-1)dx-\int_b^a(\sqrt{x}-1)dx =$
$= \int_a^c(1-\sqrt{x})dx+\int_c^b(1-\sqrt{x})dx+\int_b^a(1-\sqrt{x})dx =$
$= \int_a^b(1-\sqrt{x})dx-\int_a^b(1-\sqrt{x})dx = 0$

3. $\int_0^2(f(x)-g(x))dx = \int_0^2\left(1+3x^2-\sqrt{x}-\left(\frac{3}{2}x-\sqrt{x}\right)\right)dx = \int_0^2\left(1+3x^2-\frac{3}{2}x\right)dx =$
$= \left(\frac{2^1}{1}-\frac{0^1}{1}\right)+3\cdot\left(\frac{2^3}{3}-\frac{0^3}{3}\right)-\frac{3}{2}\cdot\left(\frac{2^2}{2}-\frac{0^2}{2}\right) = 2+8-3 = 7$